

Signatures of quantized TeV scale Black holes in scattering processes

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Abstract

In this paper we shall study the phenomenology of a doubly charged and neutral exchange black hole in an $(n + 3)$ extra-dimensional scenario, where the black hole shall be treated a normal quantum field.

1 Introduction

Extra-dimensional scenarios have been put forward in recent times as a possible solution to the so called hierarchy problem [1]. Such models typically bring down the Planck scale from their traditional high values to a scale in the TeV region. While there would be correctional terms to measurables in high energy processes coming from excitations of standard model particles, there is also the very interesting possibility that such models would give rise to the presence of black holes in the TeV region rather than the normal Planck mass scale [2]. Since TeV scale laboratory collisions are well within view, we should expect, if these models are valid, black hole production in the laboratory. The detection of such black holes would be usually studied by its decay products in such collisions.

In the classical scenario [3], one would expect a black hole to radiate through a black body radiation spectrum. This would be reflected in the decay products having large multiplicity and large transverse energy [4]. One of course has no clue at the moment whether such classical ideas would survive in a full quantum theory of gravitation which of course does not exist till now. However, this picture cannot survive if the black hole mass is close to the Planck mass. As the black hole starts radiating its mass would decrease and approach the Planck mass, where Hawking's approximation breaks down. The radiation emitted at that stage is probably highly non-thermal [5].

The situation becomes more reminiscent of atomic radiation if one accepts some current ideas regarding quantization of black hole mass. This idea of quantization was first put forward by

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Bekenstein [6] and has been dealt with by a number of authors since. If the quantization picture is correct then the ideas mentioned above, regarding radiation emitted by a black hole according to the black body radiation formula, may be expected via the correspondence principle only when the level splitting becomes very small compared with the mass. The ground state of such a system would be stable against such radiation but would probably decay into ordinary particles. If the Planck mass is indeed in the TeV region, then as we go to higher and higher values of energy in the laboratory, the ground state black hole would be the first one to be reached in a quantized black hole scenario. It is thus interesting then to think about the possible signatures for that.

In a recent publication Bilke et al. [7] have put forward the interesting idea that for such quantized black holes, for low values of quantum numbers, the black hole should be treated like any other particle, describable by a quantum field which interacts with other standard model fields. They concentrate on a doubly charged black hole and work out possible signatures of that in a possible electron-electron TeV scale scattering. We find this idea very interesting and the purpose of this note is to further consider this picture in relation to neutral systems.

The quantization formula for the black hole mass has the form

$$M_{bh}^2 = gM_P^2 \left[n \left(1 + \alpha \frac{q^2}{2n} \right) + \frac{j^2}{n} \right] \quad (1)$$

where M_P is the Planck mass, α the fine structure constant, n the radial quantum number, j the angular momentum of the black hole as constrained by

$$j^2 + \frac{\alpha q^2}{4} \leq n^2.$$

g is a pre-factor whose actual value is somewhat uncertain. We shall take the value of g as given by Khriplovich [8] and we shall see that the type of conclusions we wish to draw are independent of this particular choice. The central point about the relevance of this mass formula for high energy scattering is that it not only relates the masses and the ground state, and its excitations, but also predicts that charged and neutral black holes must exist together with unequal but correlated masses. This is completely different from the situation in a hadronic resonance where charged and neutral resonant states have only electromagnetic splittings. Not only that, as we show below, the neutral and charged black holes in the ground state will exhibit different characteristics in high energy collisions.

Consider first a doubly charged black hole in the ground state $n = 1$ which we represent as discussed above by a scalar field ϕ_c . Its interaction with electron field ψ can be written down as

$$L_{int}^c = \frac{i\kappa^c}{2} M_{bh}^c \phi_c \cdot \tilde{\psi} C \psi \quad (2)$$

where κ^c is the effective coupling constant, C is the charge conjugation matrix and M_{bh}^c is the mass of the doubly charged black hole with $q = 2$ and $j = 0$. As shown in by Bilke et al. [7], the coupling constant κ^c can be related to the Schwarzschild radius R_s in $(n + 3)$ -dimensions [9]:

$$\kappa^c = 2R_s \quad (3)$$

$$R_s = \frac{1}{\sqrt{\pi}M_P} \left[\frac{M_{bh}^c}{M_P} \left(\frac{8\Gamma(\frac{n+3}{2})}{n+2} \right) \right]^{\left(\frac{1}{n+1}\right)}. \quad (4)$$

The matrix element for the scattering process

$$e^-(p_1) + e^-(p_2) \rightarrow e^-(p_3) + e^-(p_4)$$

through the black hole of mass M_{bh}^c considered as a Breit-Wigner resonance of width Γ_{tot}^c is

$$M(p_1, p_2; p_3, p_4) = M^*(p_3, p_4) \left\{ \frac{1}{s - (M_{bh}^c + \frac{i\Gamma_{tot}^c}{2})^2} \right\} M(p_1, p_2) \quad (5)$$

$$M(p, q) = i\kappa^c M_{bh}^c \tilde{u}(p) C u(q) \quad (6)$$

where s is the centre of mass energy squared. We can readily calculate the total cross section with this amplitude. Neglecting masses in the incoming and outgoing channels, we get

$$\sigma(s) = \frac{(\kappa^c M_{bh}^c)^4 s}{16\pi} \frac{1}{|s - (M_{bh}^c + \frac{i\Gamma_{bh}^c}{2})^2|^2} \quad (7)$$

The cross section calculated above differs from the one calculated by Bilke et al. [7] in one essential respect. Unlike them we have not assumed that the resonant black hole emits particles with a black body radiation spectrum which would have necessitated multiplying the phase space at every point with a statistical factor. We feel justified in doing so since at the ground level, there is no classical Hawking radiation and therefore there is no question of particles being emitted as if they are emitted from a black body of a certain temperature. As emphasized earlier, the hypothesis we are testing is whether a black hole after emitting all the Hawking type of radiation it could and falling to the ground state can be treated like any other particle describable by a field.

The cross section given above as such is not suitable for direct comparison with experimental data since it involves an unknown quantity, namely the total width. The elastic width Γ^c can of course be easily calculated from the basic interaction given above and turns out to be

$$\Gamma^c = \frac{(\kappa^c)^2 (M_{bh}^c)^3}{8\pi}. \quad (8)$$

Bilke et al. [7] make the assumption that the total and the elastic widths are equal. We believe that this assumption is justified in the charged case but not in the neutral case and this is the key to the difference in behaviour of the two. To show this, consider an extra particle (considered spinless for simplicity) described by a field χ produced in the collision along with the electrons. The coupling of the charged black hole to the three particle state of two electrons and the scalar is effectively described by the interaction

$$L^\chi = \frac{i\kappa^c}{2M_P} M_{bh}^c \phi_c \chi \cdot \tilde{\psi} C \psi, \quad (9)$$

where the Planck mass in the denominator effectively compensates the extra dimension introduced by the scalar field in the Lagrangian. There is of course no proof that the effective Lagrangian above should be related to the Lagrangian given in equation (2) except on dimensional grounds, and the fact we are dealing with interactions at scales defined by the Planck mass. With this Lagrangian we can easily calculate the cross section for the process

$$e^- + e^- \rightarrow e^- + e^- + \chi$$

and we get

$$\sigma(e^- + e^- \rightarrow e^- + e^- + \chi) = \left[\frac{s}{48\pi^2 M_P^2} \right] \sigma(e^- + e^- \rightarrow e^- + e^-). \quad (10)$$

The factor in square brackets on the right hand-side is of course much less than one. This situation will become worse for multi-particle production and hence we find that the assumption made by Bilke et. al [7] is justified.

Consider now an electrically neutral process, like a pair of photons, producing a black hole. Like the charged case, the interaction can be defined by a Lagrangian:

$$L_{int} = \frac{\kappa}{M_{bh}} \phi \cdot F_{\mu\nu} F^{\mu\nu}, \quad (11)$$

where ϕ represents the neutral black hole field, M_{bh} its mass given by equation (1) and κ represents the coupling. Just as in the charged case, we can relate κ to the Schwarzschild radius R_s and we get an identical result $\kappa = 2R_s$. The calculation for the cross section

$$\gamma + \gamma \rightarrow \gamma + \gamma$$

through the neutral black hole proceeds exactly parallel to the charged case with identical results. The crucial difference comes about in this case because although multi-particle production will be suppressed as in the charged case there are other two particle channels available for coupling to the neutral black hole. Neglecting particle masses, all these two particle channels will be produced approximately with the same cross section and hence in this the total width is no longer going to be equal to the elastic one but will be the elastic one multiplied by the number of two particle channels. In the standard model it is easily seen there will be 28 open two body channels. The cross section for the process

$$\gamma + \gamma \rightarrow \gamma + \gamma$$

will again be given by equation (7) with this understanding of the total cross section. The total cross section in this case of course will be 28 times the elastic one. This difference between the neutral and charged cross section as outlined above dictates that the neutral one should be much more flatter with the peak occurring at a slightly lower value, as predicted by the mass formula equation (1).

Figure 1 shows our result for a value of the Planck mass equal to 1TeV and the pre-factor g taken to be $g = 0.614/\pi$. The exact position of the peak of course depends on g which as stated before has some uncertainty, but the relative position of the peak is independent of g . In figure 2 we show the same results for different values of the number of extra dimensions n .

In conclusion, we have shown that in the extra-dimensional scenario with quantized masses of the black hole, there are specific signatures which can be examined in a TeV scale scattering of particles. The chief characteristics are the necessary occurrence of a peak for neutral processes at a mass lower by a known factor from the peak position in a doubly charged scattering process. Further, characteristically, the neutral peak may be expected to be much flatter as compared to the charged one. All these seem to be realizable experimentally in the foreseeable future.

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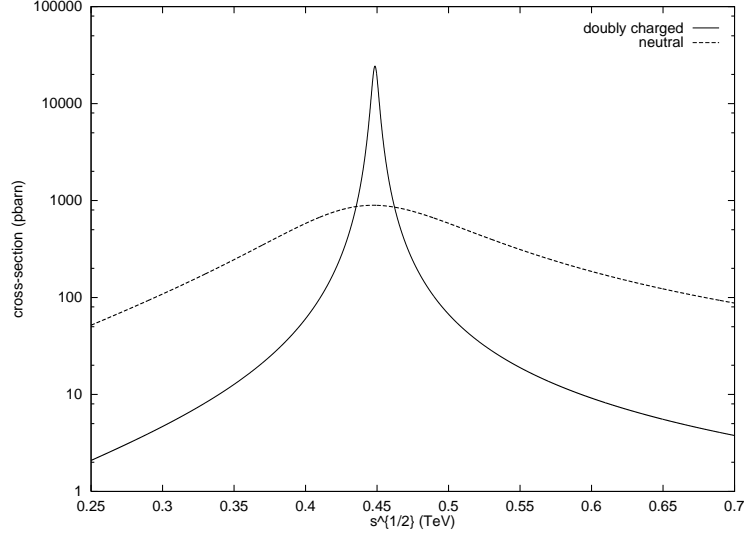


Figure 1: The total cross sections in pb for the doubly charged and neutral channels. M_P has been taken as 1TeV, whereas the pre-factor g has been taken as $0.614/\pi$.

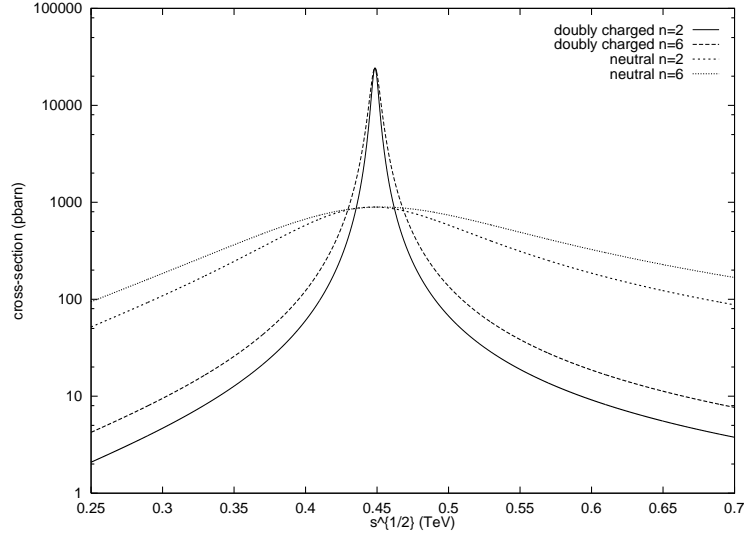


Figure 2: The total cross sections in pb for the doubly charged and neutral channels where the number of extra-dimensions n has been taken to be 2 or 6. $M_P = 1TeV$ and the pre-factor $g = 0.614/\pi$.